Inverse Trigonometry

✓ A function is invertible if it is one-one

Function	Domain	Range
$\sin\theta$	R	[-1, 1]
$\cos\theta$	R	[-1, 1]
$\tan\theta$	$R-\{n\pi+\pi/2\}$	R
$\cot\theta$	$R-\{n\pi\}$	R
$\sec\theta$	$R-\{n\pi+\pi/2\}$	$(-\infty,-1]\cup[1,\infty)$
$\operatorname{cosec}\theta$	$R-\{n\pi\}$	$(-\infty,-1]\cup[1,\infty)$

Function	Domain	Range
$\sin^{-1}x$	[-1, 1]	$[-\pi/2, \pi/2]$
$\cos^{-1}x$	[-1, 1]	$[0, \pi]$
$\tan^{-1}x$	R $(-\infty,\infty)$	$(-\pi/2, \pi/2)$
$\cot^{-1}x$	R $(-\infty,\infty)$	$(0, \pi)$
$\sec^{-1}x$	$(-\infty,-1]\cup[1,\infty)$	$[0,\pi]-\{\pi/2\}$
$\operatorname{cosec}^{-1}x$	$(-\infty,-1]\cup[1,\infty)$	$[-\pi/2, \pi/2]-\{0\}$

➢ **For principal values of "θ" and "x"**

✓ $\sin^{-1}(\sin\theta)=\theta$
✓ $\cos^{-1}(\cos\theta)=\theta$
✓ $\tan^{-1}(\tan\theta)=\theta$
✓ $\cot^{-1}(\cot\theta)=\theta$
✓ $\sec^{-1}(\sec\theta)=\theta$
✓ $\operatorname{cosec}^{-1}(\operatorname{cosec}\theta)=\theta$

✓ $\sin(\sin^{-1}x)=x$
✓ $\cos(\cos^{-1}x)=x$
✓ $\tan(\tan^{-1}x)=x$
✓ $\cot(\cot^{-1}x)=x$
✓ $\sec(\sec^{-1}x)=x$
✓ $\operatorname{cosec}(\operatorname{cosec}^{-1}x)=x$

✓ $\sin^{-1}(-x)=-\sin^{-1}x$
✓ $\cos^{-1}(-x)=\pi-\cos^{-1}x$
✓ $\tan^{-1}(-x)=-\tan^{-1}x$
✓ $\cot^{-1}(-x)=\pi-\cot^{-1}x$
✓ $\sec^{-1}(-x)=\pi-\sec^{-1}x$
✓ $\operatorname{cosec}^{-1}(-x)=-\operatorname{cosec}^{-1}x$

✓ $\sin^{-1}(1/x)=\operatorname{cosec}^{-1}x$
✓ $\cos^{-1}(1/x)=\sec^{-1}x$
✓ $\tan^{-1}(1/x)=\begin{cases}\cot^{-1}x & \text{for } x>0\\ -\pi+\cot^{-1}x, & \text{for } x<0\end{cases}$

✓ $\sin^{-1}x+\cos^{-1}x=\pi/2$
✓ $\tan^{-1}x+\cot^{-1}x=\pi/2$
✓ $\sec^{-1}x+\operatorname{cosec}^{-1}x=\pi/2$

➢ **For principal values of "x" and "y"**

✓ $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$
✓ $\tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1+xy}\right)$

$\sin^{-1}x+\sin^{-1}y=\sin^{-1}\{x\sqrt{1-y^2}+y\sqrt{1-x^2}\}$

$\sin^{-1}x-\sin^{-1}y=\sin^{-1}\{x\sqrt{1-y^2}-y\sqrt{1-x^2}\}$

$\cos^{-1}x+\cos^{-1}y=\cos^{-1}\{xy-\sqrt{1-x^2}\sqrt{1-y^2}\}$

$\cos^{-1}x-\cos^{-1}y=\cos^{-1}\{xy+\sqrt{1-x^2}\sqrt{1-y^2}\}$

➢ **For principal values of "x"**

✓ $2\sin^{-1}x = \begin{cases} -\pi - \sin^{-1}(2x\sqrt{1-x^2}), & \text{if } -1 <= x <= -1/\sqrt{2} \\ \sin^{-1}(2x\sqrt{1-x^2}), & \text{if } -1/\sqrt{2} <= x <= 1/\sqrt{2} \\ \pi - \sin^{-1}(2x\sqrt{1-x^2}), & \text{if } 1/\sqrt{2} <= x <= 1 \end{cases}$

✓ $2\cos^{-1}x = \begin{cases} 2\pi - \cos^{-1}(2x^2-1), & \text{if } -1 <= x <= 0 \\ \cos^{-1}(2x^2-1), & \text{if } 0 <= x <= 1 \end{cases}$

✓ $2\tan^{-1}x = \begin{cases} -\pi - \tan^{-1}(2x/(1-x^2)) \\ \tan^{-1}(2x/(1-x^2)) \\ \pi + \tan^{-1}(2x/(1-x^2)) \end{cases} = \begin{cases} -\pi - \sin^{-1}(2x/(1+x^2)), & \text{if } x < -1 \\ \sin^{-1}(2x/(1+x^2)), & \text{if } -1 < x < 1 \\ \pi + \sin^{-1}(2x/(1+x^2)), & \text{if } x >= 1 \end{cases}$

$= \begin{cases} -\cos^{-1}((1-x^2)/(1+x^2)), & \text{if } -\infty < x <= 0 \\ \cos^{-1}((1-x^2)/(1+x^2)), & \text{if } 0 <= x < \infty \end{cases}$

✓ $3\sin^{-1}x = \begin{cases} -\pi - \sin^{-1}(3x-4x^3), & \text{if } -1 <= x < -1/2 \\ \sin^{-1}(3x-4x^3), & \text{if } -1/2 <= x <= 1/2 \\ \pi - \sin^{-1}(3x-4x^3), & \text{if } 1/2 < x <= 1 \end{cases}$

✓ $3\cos^{-1}x = \begin{cases} 2\pi + \cos^{-1}(4x^3-3x), & \text{if } -1 <= x < -1/2 \\ 2\pi - \cos^{-1}(4x^3-3x), & \text{if } -1/2 <= x <= 1/2 \\ \cos^{-1}(4x^3-3x), & \text{if } 1/2 <= x <= 1 \end{cases}$

✓ $3\tan^{-1}x = \begin{cases} -\pi + \tan^{-1}((3x-x^3)/(1-3x^2)), & \text{if } x < -1/\sqrt{3} \\ \tan^{-1}((3x-x^3)/(1-3x^2)), & \text{if } -1/\sqrt{3} < x < 1/\sqrt{3} \\ \pi + \tan^{-1}((3x-x^3)/(1-3x^2)), & \text{if } x > 1/\sqrt{3} \end{cases}$

✓ $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}(x/\sqrt{1-x^2})$

✓ $\cos^{-1}x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}(\sqrt{1-x^2}/x)$

✓ $\tan^{-1}x = \sin^{-1}(x/\sqrt{1+x^2}) = \cos^{-1}(1/\sqrt{1+x^2})$

✓ $\cos(\sin^{-1}x) = \sin(\cos^{-1}x) = \sqrt{1-x^2}$