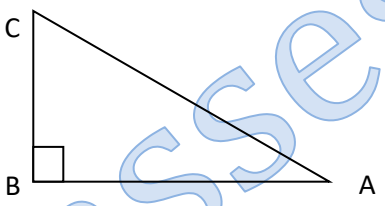
Trigonometry

Trigonometrical Ratios



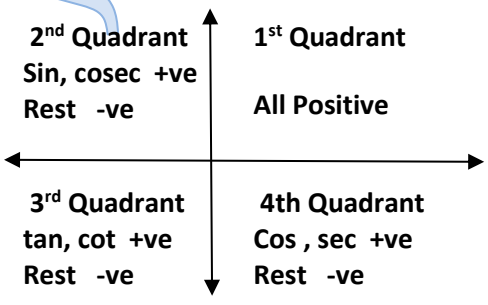
There are 6 trigonometrical ratios relating to the three sides of a right-angled triangle.

1. Sine (sin)= $\frac{\text{Perpendicular}}{\text{Hypotenuse}} \Rightarrow \sin A = \frac{BC}{AC}$ & $\sin C = \frac{AB}{AC}$
2. Cosine (Cos)= $\frac{\text{Base}}{\text{Hypotenuse}} \Rightarrow \cos A = \frac{AB}{AC}$ & $\cos C = \frac{BC}{AC}$
3. Tangent (tan)= $\frac{\text{Perpendicular}}{\text{Base}} \Rightarrow \tan A = \frac{BC}{AB}$ & $\tan C = \frac{AB}{BC}$
4. Cotangent (cot)= $\frac{\text{Base}}{\text{Perpendicular}} \Rightarrow \cot A = \frac{AB}{BC}$ & $\cot C = \frac{BC}{AB}$
5. Secant (sec)= $\frac{\text{Hypotenuse}}{\text{Base}} \Rightarrow \sec A = \frac{AC}{AB}$ & $\sec C = \frac{AC}{BC}$
6. Cosecant (cosec)= $\frac{\text{Hypotenuse}}{\text{Perpendicular}} \Rightarrow \text{cosec } A = \frac{AC}{BC}$ & $\text{cosec } C = \frac{AC}{AB}$

Relations between different Trigonometrical Ratios

1. $\sin A = \frac{1}{\text{cosec } A} \Rightarrow \sin A * \text{cosec } A = 1 \Rightarrow \text{cosec } A = \frac{1}{\sin A}$
2. $\cos A = \frac{1}{\sec A} \Rightarrow \cos A * \sec A = 1 \Rightarrow \sec A = \frac{1}{\cos A}$
3. $\tan A = \frac{1}{\cot A} \Rightarrow \tan A * \cot A = 1 \Rightarrow \cot A = \frac{1}{\tan A}$
4. $\tan A = \frac{\sin A}{\cos A}$
5. $\cot A = \frac{\cos A}{\sin A}$

Signs of the Trigonometric Ratios



Trigonometric Ratios of (-A)

$\sin(-A) = -\sin A$ $\quad \cos(-A) = \cos A$ $\quad \tan(-A) = -\tan A$

$\text{cosec}(-A) = -\text{cosec } A$ $\quad \sec(-A) = \sec A$ $\quad \cot(-A) = -\cot A$

Even Function
If f(-x)= f(x) for all x in its domain.

Odd Function
If f(-x)= -f(x) for all x in its domain.

	0	π/6	π/4	π/3	π/2	π
sin	**0**	**1/2**	**1/ √2**	**√3/2**	**1**	**0**
cos	**1**	**√3/2**	**1/ √2**	**1/2**	**0**	**-1**
tan	**0**	**1/ √3**	**1**	**√3**	**∞**	**0**
cot	**∞**	**√3**	**1**	**1/√3**	**0**	**∞**
sec	**1**	**2/√3**	**√2**	**2**	**∞**	**-1**
cosec	**∞**	**2**	**√2**	**2/√3**	**1**	**∞**

Square Relation

1. $\sin^2 A + \cos^2 A = 1$ 2. $\sec^2 A - \tan^2 A = 1$ 3. $\text{cosec}^2 A - \cot^2 A = 1$

Trigonometric ratios of sum and difference of two angles

$\sin(A+B) = \sin A \cos B + \cos A \sin B$
$\sin(A-B) = \sin A \cos B - \cos A \sin B$
$\cos(A+B) = \cos A \cos B - \sin A \sin B$
$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

$\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

$2\sin A \cos B = \sin(A+B) + \sin(A-B)$
$2\cos A \sin B = \sin(A+B) - \sin(A-B)$

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$\sin C + \sin D = 2\sin \dfrac{C+D}{2} \cos \dfrac{C-D}{2}$

$\sin C - \sin D = 2\sin \dfrac{C-D}{2} \cos \dfrac{C+D}{2}$

$\cos C + \cos D = 2\cos \dfrac{C+D}{2} \cos \dfrac{C-D}{2}$

$\cos C - \cos D = -2\sin \dfrac{C+D}{2} \sin \dfrac{C-D}{2}$

Trigonometric Ratios of Angle 2A in terms of Angle A

$\sin 2A = 2\sin A \cos A = \dfrac{2\tan A}{1+\tan^2 A}$

$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

Trigonometric Ratios of Angle 3A in terms of Angle A

$\sin 3A = 3\sin A - 4\sin^3 A$ $\cos 3A = 4\cos^3 A - 3\cos A$ $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$